Hybrid Bernoulli Filtering for Detection and Tracking of Anomalous Path Deviations

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***Abstract*—This paper presents a solution to the problem of sequential joint anomaly detection and tracking of a target subject to switching unknown path deviations. Based on a dynamic model described by Ornstein-Uhlenbeck (OU) stochastic processes, the anomaly is represented by a target (e.g., a marine vessel) that deviates from a preset route by changing its nominal mean velocity. The Random Finite Set (RFS) framework is used to represent the switching nature of target's anomalous behavior in the presence of spurious measurements and detection uncertainty. Combining these two ingredients, the problem of jointly detecting target's path deviations and estimating its kinematic state can be formulated within the Bayesian framework, and analytically solved by means of a *hybrid Bernoulli filter* that sequentially updates the joint posterior density of the unknown OU velocity input (a Bernoulli RFS) and of the target's state random vector. We illustrate the effectiveness of the proposed filter, implemented in Gaussian-mixture form, in a simulated scenario of vessel tracking for maritime traffic monitoring.**

I. Introduction

With over 80% per cent of global trade by volume, more than 3.8 billion tons of freight per year moved through the EU seaports, and approximately 395 million passenger journeys per year in the seas around the EU region, the importance and strategic function of maritime transport cannot be underemphasized. This is why nations across the globe place a high priority on ensuring maritime safety and security, and tools that help maritime surveillance to identify suspicious activity are extremely valuable. Most common security threats in maritime domain include unauthorised maritime arrivals, illegal, unreported or unregulated fishing, illegal immigration, drug smuggling, marine pollution and waste dumping, prohibited imports/exports, piracy and maritime terrorism.

Maritime surveillance data are increasingly used to achieve higher levels of situational awareness. Currently sensors used for maritime surveillance of ports and waterways include radars, infrared and video cameras, and satellite-based EO (Electro-Optical) and SAR (Synthetic Aperture Radar) imagery. Recently, however, a number of self-reporting maritime systems have been introduced, mainly for the purpose of safety in navigation and collision avoidance. For example, SAR and Vehicle Tracking System (VTS) devices have been fundamental in supporting maritime surveillance during the last decades. The most important self-reporting maritime system is the Automatic Identification System (AIS), which has been playing an increasingly important role in support to maritime surveillance and other activities such as search & rescue operations, fisheries/environment monitoring, and anomaly detection. AIS is a self-reporting messaging system, mandatory for ships over 300 gross tonnage and passenger vessels, which automatically broadcasts data about ship identity, position, velocity, and other vessel-related information. The messages transmitted by these self-reporting systems have thus become an abundant and inexpensive source of information for maritime surveillance.

Recently, anomaly detection strategies have been proposed and applied in maritime traffic monitoring [1]–[5] in order to detect unexpected ship stops or unexpected changes in course (path deviations), i.e. any vessel's *anomalous* behavior that might be related to potential suspicious activity. Most of previous work [1]–[4] typically consists of two steps: i) extraction of maritime traffic patterns via data mining of historical (training) data, and ii) anomaly detection of vessel's motion and prediction under nominal behavior by using unsupervised learning techniques. More recently, a novel maritime anomaly detector [5], relying on a hypothesis testing procedure based on the Generalized Likelihood Ratio statistic, has been proposed to reveal path deviations during AIS coverage gaps.

This paper aims to exploit the available measurements (e.g., AIS and radar data of maritime traffic) about a target following a nominal route in order to sequentially detect any anomalous behavior of the target, while jointly estimating its kinematic state. For the dynamic model, the idea is to rely on the Ornstein-Uhlenbeck stochastic process, which has been shown [6]–[8] to be a realistic model of ships' dynamics in open sea. This model allows us to represent any deviation from the nominal target's motion as an unknown input affecting the dynamics of the object under tracking; indeed, any anomalous deviation will inevitably result in an unknown contribution to the mean velocity parameter of the underlying OU process. In particular, we adopt the random set paradigm to model the target's behavior through a Bernoulli RFS that, based on the nominal or anomalous condition, will result in an empty set or a singleton, respectively. The same random set framework allows us to model the presence of a random number of spurious observations among the available data by means of a measurement RFS.

As a result, the problem of joint deviation-detection and target tracking turns out to be a *hybrid* Bayesian filtering problem [9]–[14] that aims to sequentially estimate a Bernoulli random set for the unknown long-run mean velocity input and, jointly, a random vector for the target state, given all the incoming measurements collected at each time instant. It

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is worth pointing out that, compared to the static anomaly detector proposed in [5] under AIS data unavailability, in this work we look at a similar problem but from a different perspective, i.e. we address a sequential Bayesian filtering problem where new measurements are periodically available according to the radar's scanning period or the frequency of AIS reports, and probability $p_d \in (0,1]$. This is motivated by the fact that the problem of joint detection and tracking of anomalous path deviations would ideally be dealt online, as new observations become available. Notice also that the contribution of this work, considering a single-target scenario with spurious measurements, could be further extended to the multiple-target case, by exploiting multi-target tracking algorithms (see e.g. [15], [16]).

II. Background and Problem Statement

A. Dynamic model based on Ornstein-Uhlenbeck processes

Building upon [6], we model target dynamics with OU stochastic processes. More precisely, target's velocity is modeled by means of a stochastic mean-reverting process, which tends to drift towards a central value, with an attraction that increases with the distance of the process from that central value. This (long-run mean) velocity represents the *desired* speed of the target. The key feature of this dynamic model is that the velocity process has bounded variance. The target state is represented by the four-dimensional vector $x(t) = [p(t), \dot{p}(t)]$, where $p(t)$ and $\dot{p}(t)$ are the target's position and, respectively, velocity in a two-dimensional Cartesian coordinate system. The dynamics of the target is thus ruled by the following stochastic differential equation (SDE):

$$\dot{x}(t) = Ax(t) + Bu + D\dot{w}(t), \tag{1}$$

where $u = [u_x, u_y]^T$ is the long-run mean velocity and $w(t)$ is a standard 2-D Wiener process. The matrices A, B and D are defined as

$$A = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & -\Lambda \end{bmatrix}, \quad B = \begin{bmatrix} 0_2 \\ \Lambda \end{bmatrix}, \quad D = \begin{bmatrix} 0_2 \\ \Omega \end{bmatrix}, \tag{2}$$

where 0_2 and I_2 are the 2-by-2 null and identity matrices, respectively, and $\Lambda \in \mathbb{R}^{2\times 2}$ quantifies the mean-reversion effect. If Λ has positive and distinct eigenvalues, Λ can alternatively be written as $\Lambda = T\Theta T^{-1}$, where $\Theta = \mathrm{diag}(\gamma)$. The diag operator applied to the vector $\gamma = [\gamma_1, \gamma_2]$ stores its elements on the diagonal components of a diagonal matrix. The OU process is the first moment of the solution of (1) and represents the target evolution, that can finally be written in matrix form as follows:

$$\begin{aligned} x_k = {} & \widetilde{T}\Phi(t_k - t_{k-1}, \gamma)\widetilde{T}^{-1} x(t_{k-1}) \\ & + \widetilde{T}\Psi(t_k - t_{k-1}, \gamma)T^{-1}u + w_k, \end{aligned} \tag{3}$$

where $\widetilde{T} = I_2 \otimes T$, $\otimes$ denoting the Kronecker product. The full expressions of $\Phi(t, \gamma)$ and $\Psi(t, \gamma)$ can be found in [6], [17], along with a more detailed discussion about OU processes.

Note that also a different model, i.e. the nearly-constant velocity (NCV) model, could be used to detect anomalies [1]. However, this model is inherently unsuitable for the problem under consideration, which relies on changes of the long-run mean velocity (a key parameter in OU processes, that, by contrast, is not taken into account in the NCV model) to detect and track route deviations.

B. Target under nominal/anomalous behavior

Based on the dynamic model (3) described in Section II-A using the Ornstein-Uhlenbeck process, the target under nominal/anomalous behavior can be modeled by

$$x_{k+1} = \begin{cases} \Phi^0(\delta_k, \gamma)x_k + B_k u_k^0 + w_{k+1}, & \text{under } H_0 \\ \Phi^1(\delta_k, \gamma)x_k + B_k u_k^0 + G_k u_k^1 + w_{k+1}, & \text{under } H_1 \end{cases} \tag{4}$$

where k is the time index; $\delta_k = t_k - t_{k-1}$; $x_k \in \mathbb{R}^m$ is the target kinematic state; $u_k^0 \in \mathbb{R}^q$ is a known input characterizing a target under hypothesis H_0, i.e. that follows its nominal trajectory (u_k^0 is the target's nominal long-run mean velocity at time k); $u_k^1 \in \mathbb{R}^q$ is an uknown input affecting the dynamics of the target only when it is under hypothesis H_1, i.e. when deviating from its nominal path; $\Phi^0(\delta_k, \gamma)$ and $\Phi^1(\delta_k, \gamma)$ are known state transition matrices that describe the target's state evolution, in the nominal and, respectively, anomalous cases; w_k is a random process disturbance. The target state is observed through the following model:

$$y_k = C_k x_k + v_k \tag{5}$$

where C_k is a known measurement function and v_k is a random measurement noise. It is also assumed that, due to the nature of the sensors used, e.g., in maritime surveillance, the measurement y_k is actually received with probability $p_d \in (0,1]$.

The switching anomalous behavior of the target is modelled by means of an unknown Bernoulli *velocity set* at time k, $\mathcal{U}_k^1$, which is either equal to the empty set if the target dynamics is under H_0 at time k or to the singleton $\{u_k^1\}$ otherwise, i.e.

$$\mathcal{U}_k^1 \; = \; \begin{cases} \emptyset, & \text{if the target is under } H_0 \\ \{u_k^1\}, & \text{otherwise.} \end{cases} \tag{6}$$

Due to the possible presence of spurious measurements, we define the *measurement set* at time k as

$$\mathcal{Z}_k \; = \; \mathcal{Y}_k \cup \mathcal{C}_k \tag{7}$$

where

$$\mathcal{Y}_k \; = \; \begin{cases} \emptyset, & \text{with probability } 1 - p_d \\ \{y_k\}, & \text{with probability } p_d \end{cases} \tag{8}$$

is the set of target-originated measurements, while $\mathcal{C}_k$ is the finite set of spurious observations. Note that measurement set (7) represents the information collected (or transmitted) by a generic sensor (or self-reporting system, e.g., AIS) about target's state. This data possibly includes spurious measurements of different nature (e.g., radar clutter). Clearly, in the case of AIS data, the measurement set $\mathcal{Z}_k$ will feature $\mathcal{C}_k = \emptyset$. Notice also that the unknown mean velocity term appearing in (4) under H_1 is treated, differently from the deterministic parameter u in (1), as a stochastic process $\{u_k^1\}$, independent of $x_0, \{w_{k+1}\}$ and $\{v_k\}$.

C. Random-set filtering

Here we briefly review the basic concepts of Bayesian random-set filtering [18]. A *random finite set* $\mathcal{X}$ over $\mathbb{X}$ is a random variable which takes values in the collection of all finite subsets of $\mathbb{X}$. The FISST (*FInite Set STatistics*) notion of density $f(\mathcal{X})$, also referred to as *set density*, is used to characterize RFSs. Given $f(\mathcal{X})$, the cardinality *probability mass function* $\rho(n)$ that $\mathcal{X}$ have $n \geq 0$ elements and the joint distributions $f(x_1, x_2, \ldots, x_n|n)$ over $\mathbb{X}^n$ conditional upon $|\mathcal{X}| = n$, can be written as

$$\begin{aligned} \rho(n) &= \frac{1}{n!} \int_{\mathbb{X}^n} f(\{x_1, \ldots, x_n\})\, dx_1 \cdots dx_n \\ f(x_1, x_2, \ldots, x_n|n) &= \frac{1}{n!\, \rho(n)}\ f(\{x_1, \ldots, x_n\}). \end{aligned}$$

In addition, we use the notion of FISST or *set integral* for a generic real-valued function $g(\mathcal{X})$ of an RFS $\mathcal{X}$, i.e.

$$\int g(\mathcal{X})\, \delta\mathcal{X} = g(\emptyset) + \sum_{n=1}^{\infty} \frac{1}{n!} \int g(\{x_1, \ldots, x_n\})\, dx_1 \cdots dx_n. \tag{9}$$

A specific type of RFS, i.e. the Bernoulli RFS, will be considered in this work. A Bernoulli RFS $\mathcal{X}$ on $\mathbb{X}$ has probability $1 - r$ of being empty, and probability $r \in [0, 1]$ of being a singleton $\{x\}$ whose element is distributed according to the PDF $p(x)$ defined over $\mathbb{X}$. The cardinality distribution of a Bernoulli RFS is a Bernoulli distribution with parameter r, while the set density is given by (see [18])

$$f(\mathcal{X}) = \begin{cases} 1 - r, & \text{if } \mathcal{X} = \emptyset \\ r \cdot p(x), & \text{if } \mathcal{X} = \{x\}. \end{cases} \tag{10}$$

III. Hybrid Bernoulli Filter for Joint Anomaly Detection and Tracking

In this section, we present the *Hybrid Bernoulli Random Set* (HBRS), a new type of RFS introduced by [9]–[11] in the context of resilient state estimation. The resulting hybrid Bernoulli state-space model allows us to frame and solve the problem of joint anomaly (i.e., path deviation) detection and tracking in a random set-based Bayesian framework. Following this approach, it is possible to obtain an exact recursion in terms of integral equations that generalize the Bayes and Chapman-Kolmogorov equations used for the solution of joint input-and-state estimation [19], [20] (with a switching unknown input), and standard Bernoulli filtering [18], [21]–[23] (for a system with unknown input).

A. Hybrid Bernoulli random set

Let the unknown velocity input at time k be a Bernoulli random set $\mathcal{U}_k^1 \in \mathcal{B}(\mathbb{U})$ as in (6), where $\mathcal{B}(\mathbb{U}) = \emptyset \cup \mathcal{S}(\mathbb{U})$ is a set of all finite subsets of the target velocity space $\mathbb{U} \subseteq \mathbb{R}^q$, and $\mathcal{S}$ denotes the set of all singletons (i.e., sets with cardinality 1) $\{u^1\}$ such that $u^1 \in \mathbb{U}$. Further, let $\mathbb{X} \subseteq \mathbb{R}^m$ denote the Euclidean space for the target state vector.

Based on the definition of HBRS given in [9]–[11], we consider a new state variable $(\mathcal{U}^1, x)$, defined on the hybrid space $\mathcal{B}(\mathbb{U}) \times \mathbb{X}$, which combines a Bernoulli random set, i.e. $\mathcal{U}^1$, and a random state vector x. A HBRS turns out to be fully specified by i) the probability r of $\mathcal{U}^1$ being a singleton, ii) the PDF $p^0(x)$ defined on the state space $\mathbb{X}$, and iii) the joint PDF $p^1(u^1, x)$ defined on the joint input-state space $\mathbb{U} \times \mathbb{X}$, i.e.

$$p(\mathcal{U}^1, x) = \begin{cases} (1 - r)\, p^0(x), & \text{if } \mathcal{U}^1 = \emptyset \\ r \cdot p^1(u^1, x), & \text{if } \mathcal{U}^1 = \{u^1\} \end{cases}. \tag{11}$$

The set integral over the hybrid state space can be written as

$$\int_{\mathcal{B}(\mathbb{U}) \times \mathbb{X}} p(\mathcal{U}^1, x) \delta\mathcal{U}^1\, dx = \int p(\emptyset, x)\, dx + \iint p(\{u^1\}, x)\, du^1\, dx \tag{12}$$

where the set integration with respect to $\mathcal{U}^1$ is defined according to (9), while integration with respect to x is ordinary. It can be easily verified that $p(\mathcal{U}^1, x)$ integrates to one by substituting (11) in (12), $p^0(x)$ and $p^1(u^1, x)$ being conventional probability density functions on $\mathbb{X}$ and $\mathbb{U} \times \mathbb{X}$, respectively. This means that (11) is a FISST probability density for the HBRS $(\mathcal{U}^1, x)$. This density will be referred to as *hybrid Bernoulli density* in the remainder of the paper.

The HBRS $(\mathcal{U}^1, x)$ can be corrected and predicted in a recursive fashion so as to form a *hybrid Bernoulli filter* (HBF).

B. Correction step

A complete derivation of the correction step of the hybrid Bernoulli filter for discrete-time systems with direct feedthrough can be found in [10]. Here we summarize the main results derived for a system with no direct feedthrough, as the one described by model (4)-(5). The measurement set $\mathcal{Z}_k$ is the union of two independent random sets, i.e. $\mathcal{Y}_k$ and $\mathcal{C}_k$. As we can see from (8), $\mathcal{Y}_k$ is a Bernoulli random set with cardinality 0 or 1 conditional upon the reception of y_k. On the other hand, we assume no prior knowledge on the number of false measurements, i.e. on the cardinality of $\mathcal{C}_k$. As a consequence, $\rho(n)$ is modeled as an uninformative distribution; such an assumption leads to the following FISST probability density of spurious-only measurements:

$$\eta(\mathcal{C}_k) \propto |\mathcal{C}_k|! \prod_{y_k \in \mathcal{C}_k} c(y_k) \tag{13}$$

where the PDF $c(y_k)$ describes the distribution of false data on the measurement space $\mathbb{Y}$. In the case of no prior knowledge assumed on such a distribution, $c(y_k)$ can be modeled as an uninformative PDF over $\mathbb{Y}$. By using (13), the likelihood of the measurement set $\mathcal{Z}_k$ can be obtained as stated in the following lemma. This result can be derived from Lemma 1 in [11] and by noticing that in this case the likelihood is independent of $\mathcal{U}^1$, since in (5) there is no direct feedthrough of the unknown velocity input to the output.

Lemma 1: Let $|\mathcal{Z}_k| = \xi$, then the multi-object likelihood function $\lambda(\mathcal{Z}_k|x_k)$ can be written as

$$\lambda(\mathcal{Z}_k|x_k) = \eta(\mathcal{Z}_k) \left[1 - p_d + p_d \sum_{y_k \in \mathcal{Z}_k} \frac{\ell(y_k|x_k)}{\xi\, c(y_k)} \right]. \tag{14}$$

Note that the first term on the right-hand side corresponds to the event of no reception of the target-originated measurement, i.e. $\mathcal{C}_k = \mathcal{Z}_k$, while the second term accounts for the union of the two disjoint events that one observation of $\mathcal{Z}_k$ originated from the target while the remaining $\xi - 1$ are spurious measurements, i.e. $\mathcal{C}_k = \mathcal{Z}_k \setminus \{y_k\}$ for any $y_k \in \mathcal{Z}_k$. Notice also that, by assuming a Poisson-distributed number of spurious measurements, (14) reduces to the standard multi-object likelihood function with clutter (see, e.g., [21]) with ξ, in this case, denoting the expected cardinality of clutter.

Using the above measurement model, exact correction equations of the Bayesian random set filter for joint anomaly detection and target tracking can be obtained from the following application of the Bayes rule:

$$p(\mathcal{U}^1_{k-1}, x_k|\mathcal{Z}^k) = \frac{\lambda(\mathcal{Z}_k|x_k)\, p(\mathcal{U}^1_{k-1}, x_k|\mathcal{Z}^{k-1})}{p(\mathcal{Z}_k|\mathcal{Z}^{k-1})} \tag{15}$$

where $\lambda(\mathcal{Z}_k|x_k)$ is given by (14). It can be noticed that, since model (4)-(5) describes a system with no direct feedthrough, the first measurement providing information about $\mathcal{U}^1_{k-1}$ is $\mathcal{Z}_k$ [19]. This means that input estimation under H_1 is delayed by one time unit and, thus, we need to sequentially update the hybrid Bernoulli density $p(\mathcal{U}^1_{k-1}, x_k|\mathcal{Z}^k)$. From now on, the notation $\langle a, b\rangle = \int a(x)b(x)\mathrm{d}x$ will be used for the inner product of two functions.

Theorem 1: Assume that the prior density at time k is given by the following hybrid Bernoulli density:

$$\begin{aligned} & p(\mathcal{U}^1_{k-1}, x_k|\mathcal{Z}^{k-1}) \\ &= \begin{cases} (1 - r_{k|k-1}) p^0_{k|k-1}(x_k), & \text{if } \mathcal{U}^1_{k-1} = \emptyset \\ r_{k|k-1} \cdot p^1_{k|k-1}(u^1_{k-1}, x_k), & \text{if } \mathcal{U}^1_{k-1} = \{u^1_{k-1}\} \end{cases} . \end{aligned} \tag{16}$$

Then, given the measurement RFS $\mathcal{Z}_k$ in (7), the posterior density at time k is also hybrid Bernoulli, given by

$$p(\mathcal{U}^1_{k-1}, x_k|\mathcal{Z}^k) = \begin{cases} (1 - r_{k|k})\, p^0_{k|k}(x_k) & \text{if } \mathcal{U}^1_{k-1} = \emptyset \\ r_{k|k} \cdot p^1_{k|k}(u^1_{k-1}, x_k) & \text{if } \mathcal{U}^1_{k-1} = \{u^1_{k-1}\} \end{cases} \tag{17}$$

with

$$r_{k|k} = \frac{1 - p_d\,(1 - \Delta_1)}{1 - p_d(1 - \Delta_0 + r_{k|k-1}\Delta)}\, r_{k|k-1} \tag{18}$$

$$p^0_{k|k}(x_k) = \frac{1 - p_d \left[1 - \displaystyle\sum_{y_k \in \mathcal{Z}_k} \frac{\ell(y_k|x_k)}{\xi\, c(y_k)}\right]}{1 - p_d\,(1 - \Delta_0)} \times p^0_{k|k-1}(x_k) \tag{19}$$

$$p^1_{k|k}(u^1_k, x_k) = \frac{1 - p_d \left[1 - \displaystyle\sum_{y_k \in \mathcal{Z}_k} \frac{\ell(y_k|x_k)}{\xi\, c(y_k)}\right]}{1 - p_d\,(1 - \Delta_1)} \times p^1_{k|k-1}(u^1_{k-1}, x_k) \tag{20}$$

and

$$\Delta_0 \triangleq \sum_{y_k \in \mathcal{Z}_k} \frac{\left\langle \ell(y_k|x_k), p^0_{k|k-1}(x_k) \right\rangle}{\xi\, c(y_k)} \tag{21}$$

$$\Delta_1 \triangleq \sum_{y_k \in \mathcal{Z}_k} \frac{\left\langle \ell(y_k|x_k), p^1_{k|k-1}(u^1_{k-1}, x_k) \right\rangle}{\xi\, c(y_k)} \tag{22}$$

and $\Delta \triangleq \Delta_0 - \Delta_1$. □

Equations (17)-(20) fully specify the correction step of the hybrid Bernoulli filter for joint anomaly-detection and tracking. Notice that if $p_d = 1$ and $r_{k|k-1} = 1$, then $r_{k|k} = 1$ follows from (18). If we further assume $\mathcal{C}_k = \emptyset$, then (20) reduces to the standard Bayes filter correction solving the *Joint Input and State Estimation* problem without direct feedthrough [19], [20]. Under the same assumptions, if $r_{k|k-1} = 0$, then $r_{k|k} = 0$, and thus (19) reduces to the standard Bayes filter correction (with no unknow input).

C. Prediction step

In order to derive the prediction equations of the Bayesian hybrid Bernoulli filter for joint anomaly detection and target tracking, it is reasonable to assume that the joint transitional density of $(\mathcal{U}^1, x)$ at time $k+1$ takes the form

$$p(\mathcal{U}^1_k, x_{k+1}|\mathcal{U}^1_{k-1}, x_k) = p(x_{k+1}|\mathcal{U}^1_{k-1}, x_k)\, p(\mathcal{U}^1_k|\mathcal{U}^1_{k-1}) \tag{23}$$

which follows from considering the unknown velocity set as independent of the system state, as assumed in Section II-B. From the dynamic model (4), one has

$$p(x_{k+1}|\mathcal{U}^1_{k-1}, x_k) = p(x_{k+1}|x_k) \tag{24}$$

where $p(x_{k+1}|x_k)$ is a known Markov transition PDF.

The transitional density $p(\mathcal{U}^1_k|\mathcal{U}^1_{k-1})$ is ruled by two design parameters p_b and p_s, chosen in such a way that the event of the target deviating at time k is more probable when the target is already detouring at time $k-1$. To this end, we assume that i) a target under nominal behavior at time $k-1$, will start a path deviation, characterized by u^1_k, during the sampling interval δ_k with probability p_b; ii) when, instead, the target is already deviating at time $k-1$ (i.e., $\mathcal{U}^1_{k-1}$ is a singleton), the anomalous action will carry on from $k-1$ to k with probability p_s, and transition model

$$u^1_k = \alpha^{\delta_k} u^1_{k-1} + (1-\alpha)^{\delta_k} \zeta_k \tag{25}$$

where $\alpha \in [0, 1]$ and $\zeta_k \sim \mathcal{N}(0, \Xi), \Xi \triangleq \sigma^2_\zeta I$ being a zero-mean white noise. The tuning of probabilities p_b and p_s can lead to different filter's performance. As an example, the lower is p_b (probability of the target under H_0 at time $k-1$ starting a new path deviation at time k) the more prudent will be the filter in announcing the beginning of a new target's deviation. Analogously, the higher is p_s (probability of the target under H_1 at time $k-1$ continuing deviating from the nominal trajectory at time k) the more reluctant will be the filter in declaring that the target switched behavior and stopped detouring. To sum up, under the above assumptions, the dynamics of $\mathcal{U}_{k-1}^1$ can be modeled as a Bernoulli Markov process described by

$$p(\mathcal{U}_k^1|\emptyset) = \begin{cases} 1-p_b, & \text{if } \mathcal{U}_k^1 = \emptyset \\ p_b\, p(u_k^1), & \text{if } \mathcal{U}_k^1 = \{u_k^1\} \end{cases} \quad (26)$$

$$p(\mathcal{U}_k^1|\{u_{k-1}^1\}) = \begin{cases} 1-p_s, & \text{if } \mathcal{U}_k^1 = \emptyset \\ p_s\, p(u_k^1|u_{k-1}^1), & \text{if } \mathcal{U}_k^1 = \{u_k^1\} \end{cases} \quad (27)$$

where $p(u_k^1)$ is a PDF representing the prior knowledge on the unknown velocity input u_k^1 which characterizes a new deviation started during the interval δ_k. As typically done in the literature on unknown input estimation [19], [20], an uninformative PDF (e.g., uniform over the input space $\mathbb{U}$) can be used to model $p(u_k^1)$ when the velocity input u^1 is completely unknown. This choice, in the Bayesian framework, leads to a maximum-likelihood (ML) estimation of the unknown input.

Under the above model, an exact recursion for the predicted density can be obtained, as stated in the following theorem.

Theorem 2: Let $p(\mathcal{U}_{k-1}^1, x_k|\mathcal{Z}^k)$ be the posterior hybrid Bernoulli density at time k of the form (17), completely specified by the triplet $\left(r_{k|k}, p_{k|k}^0(x_k), p_{k|k}^1(u_{k-1}^1, x_k)\right)$, then the predicted density, also hybrid Bernoulli, takes the form

$$p(\mathcal{U}_k^1, x_{k+1}|\mathcal{Z}^k) \quad (28)$$
$$= \begin{cases} (1-r_{k+1|k})\, p_{k+1|k}^0(x_{k+1}) & \text{if } \mathcal{U}_k^1 = \emptyset \\ r_{k+1|k} \cdot p_{k+1|k}^1(u_k^1, x_{k+1}) & \text{if } \mathcal{U}_k^1 = \{u_k^1\} \end{cases}$$

where

$$r_{k+1|k} = (1-r_{k|k})\, p_b + r_{k|k}\, p_s \quad (29)$$

$$p_{k+1|k}^0(x_{k+1}) = \frac{(1-r_{k|k})(1-p_b)\, p_{k+1|k}(x_{k+1}|\emptyset)}{1-r_{k+1|k}} + \frac{r_{k|k}(1-p_s)\, p_{k+1|k}(x_{k+1}|\{u_k^1\})}{1-r_{k+1|k}} \quad (30)$$

$$p_{k+1|k}^1(u_k^1, x_{k+1}) = \frac{(1-r_{k|k})\, p_b\, p_{k+1|k}(x_{k+1}|\emptyset)\, p(u_k^1)}{r_{k+1|k}} + \frac{r_{k|k}\, p_s\, p_{k+1|k}(u_k^1, x_{k+1}|\{u_k^1\})}{r_{k+1|k}} \quad (31)$$

and

$$p_{k+1|k}(x_{k+1}|\emptyset) = \left\langle p(x_{k+1}|x_k), p_{k|k}^0(x_k) \right\rangle \quad (32)$$

$$p_{k+1|k}(x_{k+1}|\{u_k^1\}) = \left\langle p(x_{k+1}|x_k), p_{k|k}^1(u_{k-1}^1, x_k) \right\rangle \quad (33)$$

$$p_{k+1|k}(u_k^1, x_{k+1}|\{u_k^1\}) = \left\langle p(x_{k+1}|x_k)\, p(u_k^1|u_{k-1}^1), p_{k|k}^1(u_{k-1}^1, x_k) \right\rangle \quad (34)$$

□

As we can see from (29), the target is predicted to be under anomalous behavior at time k if either a pre-existing anomaly persists after time $k-1$, or a new u_k^1 enters into effect at time k. It is also evident in (31) that the prediction step involves two separate terms which account for the start of a new anomalous path deviation and, respectively, for the continuation of an already-started anomalous behavior. This two-component expression for the prior density is similar to the one characterizing the standard Bernoulli filter [21].

Analogously, from (30) we notice that the target is predicted to be under nominal behavior at time k if either no new deviations are initiated, or the previously existing (*legacy*) anomaly ends and the target switches back to nominal motion during interval δ_k.

D. Practical considerations about hybrid Bernoulli filtering

Making use of the results in Theorem 1 and Theorem 2, it is possible to practically perform simultaneous anomaly detection and tracking from the available current hybrid Bernoulli density as described below.

Given the HBRS density $p(\mathcal{U}_{k-1}^1, x_k|\mathcal{Z}^k)$, containing the available information on $(\mathcal{U}_{k-1}^1, x_k)$ after measurements $\mathcal{Z}_{1:k} \triangleq \cup_{i=1}^k \mathcal{Z}_i$ have been processed, then the current density can be updated at each time k by means of the following steps.

1) *Prediction:* Compute $p(\mathcal{U}_k^1, x_{k+1}|\mathcal{Z}^k)$ from the posterior density $p(\mathcal{U}_{k-1}^1, x_k|\mathcal{Z}^k)$ by exploiting the OU dynamic model according to the results of Theorem 2.
2) *Correction:* Compute $p(\mathcal{U}_k^1, x_{k+1}|\mathcal{Z}^{k+1})$ from the prior density $p(\mathcal{U}_k^1, x_{k+1}|\mathcal{Z}^k)$ by exploiting measurements $\mathcal{Z}_{1:k+1}$ according to the results of Theorem 1.
3) *Anomaly detection & tracking:* Perform anomaly detection using $r_{k+1|k+1}$ from the available current hybrid Bernoulli density $p(\mathcal{U}_k^1, x_{k+1}|\mathcal{Z}^{k+1})$, i.e. based on the MAP decision rule, assign $\hat{\mathcal{U}}_k^1 \neq \emptyset$ (the target is under anomalous behavior at time k, i.e. under H_1) if and only if $\text{Prob}(\mathcal{U}_k^1 \neq \emptyset|\mathcal{Z}^{k+1}) > \text{Prob}(\mathcal{U}_k^1 = \emptyset|\mathcal{Z}^{k+1})$. Lastly, perform target tracking by maximizing $p_{k+1|k+1}^1(u_k^1, x_{k+1})$ to obtain a MAP estimate of target's kinematic state and an ML estimate of the unknown velocity input (if under H_1), or by maximizing $p_{k+1|k+1}^0(x_{k+1})$ to obtain a MAP estimate of target's state (if under H_0).

It is also worth mentioning that, likewise standard Bernoulli filters, the proposed hybrid Bernoulli filter in general does not admit an exact closed-form solution since the posterior PDFs of the state to be estimated are infinite-dimensional. Nevertheless, for linear Gaussian models of the form (4)-(5) the posterior PDFs in (19) and (20) can be finitely parameterized by means of linear combinations of Gaussian components whose growing number can be limited via simple pruning and merging procedures like the ones described in [24] in order to enable on-line processing. This Gaussian-mixture approach can be generalized to nonlinear models and/or non-Gaussian noises.

IV. Vessel Tracking Case-Study

The performance of the proposed hybrid Bernoulli filter, implemented via Gaussian mixtures, has been tested on a simulated scenario concerning maritime traffic monitoring.

In this example, the objective is to detect and track route deviations of a vessel navigating along a nominal path with mean velocity $u^0 = [10, 0]^T m/s$. In particular, we assume that the evolution of vessel's kinematic state (i.e. position and velocity) over time follows the discrete-time OU model (4) with transition matrices $\Phi^0 = \Phi^1$, output matrix $C = I_m$, $m = 4$, and input matrix $G = B$.

For this case-study, we used the following parameters: mean-reverting rate of the underlying OU process $\gamma_1 = \gamma_2 = 0.9 \times 10^{-2}$, constant sampling interval $\delta = 30$ minutes, process noise covariance matrix $\Omega = \text{diag}(10^{-2}, 10^{-2}, 10^{-4}, 10^{-4})$, measurement noise covariance matrix $R = \text{diag}(10^3, 10^3, 10^1, 10^1)$, probabilities of new anomaly and anomaly-survival $p_b = 0.001$ and $p_s = 0.1$. $p_d = 0.9$, $r_{1|0} = 0.4$, $x_0 = 0$. Both densities $p^0(\cdot)$ and $p^1(\cdot$ have been initialized as single Gaussian components with fi guess mean $\hat{x}^0_{1|0} = [0, 0, 10, 0]^T$, $\hat{x}^1_{1|0} = [100, 100, 20, -2($ and covariance $P^0_{1|0} = P^1_{1|0} = \text{diag}(10^4, 10^4, 10^2, 10^2)$. Th we set $\tilde{u}^1_{1|0} = [20, -20]^T$ and $\tilde{P}^{u^1}_{1|0} = 10^3 I_2$ as mean a covariance of the a-priori PDF $p(u^1_k)$.

The spurious measurements have been modeled as u formly distributed over the surveillance area, i.e. over the int val $[-10, 2 \times 10^3]\, Km$, for the x and y position coordinat and over the interval $[-50, 50]\, m/s$ for the velocity x- a y-components. False measurements have been generated w average number $d = 0.5$. Finally, pruning and merging thres. olds for the GM implementation have been set as $\mu_p = 10^{-3}$ and $\mu_m = 3$, respectively.

The vessel starts deviating from the nominal route at time step $k = 11$ and continue its detour until time $k = 26$. As shown in Fig. 1, the proposed filter promptly detects the anomalous path deviation, as we can see from the estimated probability of anomaly's existence $r_{k|k}$ (averaged over 100 Monte Carlo trials) which takes the unitary value as soon as the target starts its deviation and an abrupt change in velocity takes place (time step $k = 11$). Furthermore, we can see that once the deviation is over, the estimated probability $r_{k|k}$ goes back to zero within a short time (it goes below 0.5 with a one-step delay). The performance of the hybrid Bernoulli filter in terms of long-run mean velocity reconstruction under nominal/anomalous target behavior is shown in Fig. 2. The filter keeps tracking the velocity vector with high accuracy in correspondence of nominal target's motion; moreover, even under the unknown target deviation, the HBF succeeds in remaining locked on to the target's change in velocity with only a short time lag. Finally, Fig. 3 shows the RMSE (averaged over 100 Monte Carlo runs) relative to the estimate $u^1_{k|k}$ of the unknown velocity input characterizing the target's motion under the hypothesis of anomalous behavior H_1.

V. Conclusion

Building upon the hybrid Bernoulli filtering framework introduced in [9]–[11] for discrete-time systems with direct feedthrough and exploiting Ornstein-Uhlenbeck stochastic processes to represent target's dynamics [6], we proposed a novel approach to address the problem of joint anomaly detection and tracking in the face of unknown path deviations of a target supposed to follow a preset nominal route. Random finite sets have been exploited in order to model the switching nature of target's anomalous behavior, and the resulting hybrid Bernoulli filtering problem in the case of no direct feedthrough on the output has been formulated and solved. The proposed tools have shown promising results for future

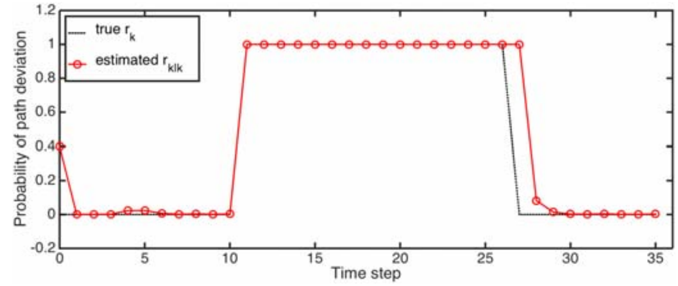

Fig. 1: True and estimated probability of target's path deviation. The estimated $r_{k|k}$ is averaged over 100 Monte Carlo runs.

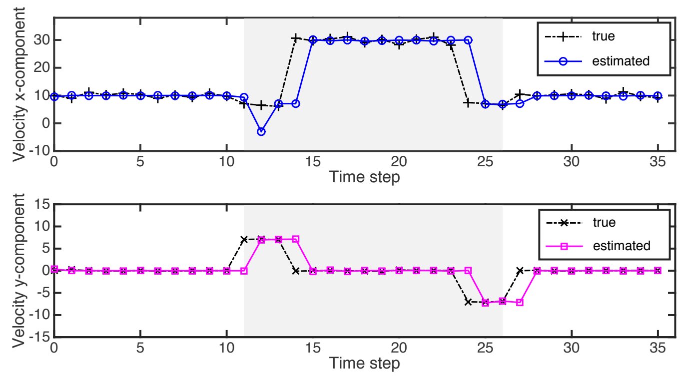

Fig. 2: Single-run performance of the hybrid Bernoulli filter in terms of long-run mean velocity (x and y components) reconstruction under both nominal and anomalous target behavior. The gray band marks the interval $[11, 26]$ ranging from the start to the finish time step of target's deviation.

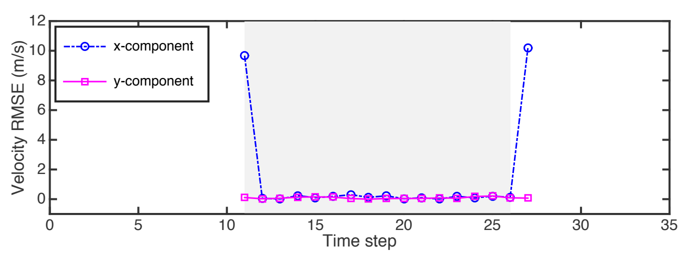

Fig. 3: Performance of the hybrid Bernoulli filter in terms of RMSE of the unknown OU velocity estimation when the target is assumed under anomalous behavior (hypothesis H_1) by the filter.

application in maritime traffic surveillance. Future work will investigate possible extensions to i) a multiple-model HBF approach, inspired by [14], for simultaneous estimation of the OU parameters governing the target's dynamics; ii) a multiple-target scenario, by exploiting multi-target algorithms [15], [16]; iii) the design of resilient monitoring systems that can perform effectively also in the presence of counterfeit measurements, e.g. generated via AIS spoofing attacks [25].

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REFERENCES

[1] B. Ristic, B. L. Scala, M. Morelande, and N. Gordon, "Statistical analysis of motion patterns in AIS data: Anomaly detection and motion prediction," in *11th International Conference on Information Fusion*, pp. 1–7, 2008.

[2] R. O. Lane, D. A. Nevell, S. D. Hayward, and T. W. Beaney, "Maritime anomaly detection and threat assessment," in *13th International Conference on Information Fusion*, pp. 1–8, 2010.

[3] K. Kowalska and L. Peel, "Maritime anomaly detection using gaussian process active learning," in *15th International Conference on Information Fusion*, pp. 1164–1171, 2012.

[4] M. Vespe, I. Visentini, K. Bryan, and P. Braca, "Unsupervised learning of maritime traffic patterns for anomaly detection," in *9th IET Data Fusion Target Tracking Conference*, pp. 1–5, 2012.

[5] E. d'Afflisio, P. Braca, L. M. Millefiori, and P. Willett, "Maritime anomaly detection based on mean-reverting stochastic processes applied to a real-world scenario," in *21st International Conference on Information Fusion*, Cambridge, UK, 2018.

[6] L. M. Millefiori, P. Braca, K. Bryan, and P. Willett, "Modeling vessel kinematics using a stochastic mean-reverting process for long-term prediction," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 52, no. 5, pp. 2313–2330, 2016.

[7] L. M. Millefiori, P. Braca, and P. Willett, "Consistent estimation of randomly sampled Ornstein-Uhlenbeck process long-run mean for long-term target state prediction," *IEEE Signal Processing Letters*, vol. 23, no. 11, pp. 1562–1566, 2016.

[8] G. Vivone, L. M. Millefiori, P. Braca, and P. Willett, "Performance assessment of vessel dynamic models for long-term prediction using heterogeneous data," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 55, no. 11, pp. 6533–6546, 2017.

[9] N. Forti, G. Battistelli, L. Chisci, and B. Sinopoli, "A Bayesian approach to joint attack detection and resilient state estimation," in *Proc. 55th IEEE Conference on Decision and Control*, pp. 1192–1198, Las Vegas, Nevada, USA, 2016.

[10] N. Forti, G. Battistelli, L. Chisci, and B. Sinopoli, "Joint attack detection and secure state estimation of cyber-physical systems," Tech. Rep., 2016 [Online]. Available: http://arxiv.org/abs/1612.08478.

[11] N. Forti, G. Battistelli, L. Chisci, and B. Sinopoli, "Resilient state estimation against unknown switching inputs and extra packet injections," under review.

[12] N. Forti, G. Battistelli, L. Chisci, S. Li, B. Wang, and B. Sinopoli, "Distributed joint attack detection and secure state estimation," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 4, no. 1, pp. 96–110, 2018.

[13] N. Forti, G. Battistelli, L. Chisci, and B. Sinopoli, "Worst-case analysis of joint attack detection and resilient state estimation," in *IEEE 56th Conference on Decision and Control*, pp. 182–188, Melbourne, Australia, 2017.

[14] N. Forti, G. Battistelli, L. Chisci, and B. Sinopoli, "Secure state estimation of cyber-physical systems under switching attacks," *IFAC-PapersOnLine. 20th IFAC World Congress*, vol. 50, no. 1, pp. 4979–4986, Toulouse, France, 2017.

[15] Y. Bar-Shalom and X. Li, *Multitarget-multisensor Tracking: Principles and Techniques*. Yaakov Bar-Shalom, 1995.

[16] F. Meyer, T. Kropfreiter, J. L. Williams, R. Lau, F. Hlawatsch, P. Braca, and M. Z. Win, "Message passing algorithms for scalable multitarget tracking," *Proceedings of the IEEE*, vol. 106, no. 2, pp. 221–259, 2018.

[17] P. Coscia, P. Braca, L. M. Millefiori, F. A. N. Palmieri, and P. Willett, "Multiple Ornstein-Uhlenbeck processes for maritime traffic graph representation," *IEEE Trans. on Aerospace and Electronic Systems*, 2018.

[18] R. P. S. Mahler, *Statistical multisource multitarget information fusion*. Norwood, MA, USA: Artech House, Inc., 2007.

[19] H. Fang, R. A. De Callafon, and J. Cortés, "Simultaneous input and state estimation for nonlinear systems with applications to flow field estimation," *Automatica*, vol. 49, no. 9, pp. 2805–2812, 2013.

[20] S. Gillijns and B. D. Moor, "Unbiased minimum-variance input and state estimation for linear discrete-time systems," *Automatica*, vol. 43, no. 1, pp. 111–116, 2007.

[21] B. Ristic, B.-T. Vo, B.-N. Vo, and A. Farina, "A tutorial on Bernoulli filters: theory, implementation and applications," *IEEE Transactions on Signal Processing*, vol. 61, no. 13, pp. 3406–3430, 2013.

[22] B.-T. Vo, D. Clark, B.-N. Vo, and B. Ristic, "Bernoulli forward-backward smoothing for joint target detection and tracking," *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4473–4477, 2011.

[23] B.-T. Vo, C. M. See, N. Ma, and W. T. Ng, "Multi-sensor joint detection and tracking with the Bernoulli filter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 2, pp. 1385–1402, 2012.

[24] B.-N. Vo and W. K. Ma, "The Gaussian mixture probability hypothesis density filter," *IEEE Transactions on Signal Processing*, vol. 54, no. 11, pp. 4091–4104, 2006.

[25] F. Katsilieris, P. Braca, and S. Coraluppi, "Detection of malicious AIS position spoofing by exploiting radar information," in *16th Int. Conf. on Information Fusion*, pp. 1196–1203, Istanbul, Turkey, 2013.