ANOMALY DETECTION AND TRACKING BASED ON MEAN–REVERTING PROCESSES WITH UNKNOWN PARAMETERS

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ABSTRACT

Piecewise mean-reverting stochastic processes have been recently proposed and validated as an effective model for long-term object prediction. In this paper, we exploit the Ornstein-Uhlenbeck (OU) dynamic model to represent an anomaly as any deviation of the long-run mean velocity from the nominal condition. This amounts to modeling the anomaly as an unknown switching control input that can affect the dynamics of the object. Under this model, the problem of joint anomaly detection and tracking can be addressed within the Bayesian random set framework by means of a hybrid Bernoulli filter (HBF) that sequentially estimates a Bernoulli random set (empty under nominal behavior) for the unknown long-run mean velocity, and a random vector for the kinematic state of the object. An additional challenge is represented by the fact that two extra parameters, i.e. the reversion rate and the noise covariance of the underlying OU process, need to be specified for Bayes-optimal prediction. We propose a multiple-model adaptive filter (MMA-HBF) for anomaly detection, tracking and simultaneous estimation of the OU unknown parameters. The effectiveness of these tools is demonstrated on a simulated maritime scenario.

Index Terms— Anomaly detection; mean-reverting Ornstein-Uhlenbeck stochastic process; random finite sets; multiple-model approach; hybrid Bernoulli filter.

1. INTRODUCTION

Anomaly detection strategies have been recently proposed and applied in maritime traffic monitoring [1–8] in order to detect unexpected ship stops or unexpected deviations from standard routes, i.e. any vessel's *anomalous* behavior that might be related to potential suspicious activity. Unlike previous work on maritime situational awareness, here the anomaly detection problem is addressed relying on a novel Bayesian random finite set (RFS) filter [9] that builds on the changes in the OU process long-term mean velocity of the object. The Ornstein-Uhlenbeck process has been shown [10–12] to better model the behavior of real-world targets, such as marine vessels, with respect to conventional models including the nearly-constant velocity (NCV) [13]. In this framework, the use of the OU model turns out to be a valuable tool to represent any deviation from the nominal motion as an unknown input affecting the object dynamics.

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We can take advantage of recent results on *hybrid Bernoulli filtering* [14–18], developed for simultaneous input and state estimation of systems affected by switching unknown inputs, to frame the posed problem within a Bayesian framework. The hybrid Bernoulli filter is an attractive Bayesian random set approach to sequentially estimate a Bernoulli RFS for the unknown input (here represented by any deviation from the nominal long-run mean velocity), and a random vector for the system state in the presence of clutter, noise, and miss-detections. The HBF admits a closed-form solution for linear Gaussian models. However, this model is not general enough to accommodate object dynamics that switch between different models. In fact, unlike the NCV, the OU process is governed by a set of parameters: the long-run mean velocity, the reversion rate, and the process noise covariance. In order to perform optimal prediction in the Bayesian sense, one would need to know such process parameters, which are clearly unknown in practical applications. The parameters of the OU process can be estimated from the data in a batch fashion as presented in [10]. However, this requires full knowledge of the process realization.

In this paper, we propose to estimate the OU process parameters on the fly, relying on a multiple-model (MM) architecture [19], [20]. In multiple-model approaches, a bank of filters, each matched to a different mode of operation, runs in parallel. Then, the overall state estimate is obtained as a weighted sum of the estimates from each filter. Our MM framework consists of the parallel implementation of a bank of hybrid Bernoulli filters, each matched to a different mode representing the model of motion (characterized by specific OU process parameters) that the target is currently traveling under. The object state variable is thus extended to include a new multiple-model parameter which is incorporated into the RFS framework to form general MM prediction and correction expressions of the generic HBF equations. This leads to the development of a fully adaptive filter which aims at deciding which model is the best representation of the current dynamics of the object without requiring any prior knowledge of two key parameters of the underlying OU process, i.e., the reversion rate and the noise level. Based on this decision, the MMA-HBF seeks to detect any anomalous deviation of the object as well as estimate its kinematic state.

2. LONG-TERM OBJECT PREDICTION

A novel method [10], based on the Ornstein-Uhlenbeck mean-reverting stochastic process, has been lately proposed to address the problem of long-term object prediction. This model has been shown [10–12] to be a realistic model of ships' movements in open sea which reduces by orders of magnitude the uncertainty region related to the predicted object position with respect to state-of-the-art models. The main difference between the OU process and other well-established models is a feedback loop which ensures that the velocity of the object does not diverge over time, but is instead bounded around a finite value representing the desired (cruise) velocity of the target.

2.1. Mean-reverting dynamic model

Building upon [10], we model the object dynamics with OU stochastic processes. The object velocity is then represented by a stochastic mean-reverting process which tends to drift towards a long-run mean value with an attraction proportional to the deviation of the process from this value. Let the target state be represented by $x(t) = [p(t), \dot{p}(t)]$, where $p(t)$ and $\dot{p}(t)$ are the target's position and, respectively, velocity in a two-dimensional Cartesian coordinate system. Then, the object's dynamics are described by the following stochastic differential equation:

$$\dot{x}(t) = Ax(t) + Bu + D\dot{w}(t), \tag{1}$$

where $u = [u_x, u_y]^T$ is the long-run mean velocity and $w(t)$ is a standard 2-D Wiener process. The matrices A, B and D are defined as

$$A = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & -\Lambda \end{bmatrix}, \quad B = \begin{bmatrix} 0_2 \\ \Lambda \end{bmatrix}, \quad D = \begin{bmatrix} 0_2 \\ \Omega \end{bmatrix}, \tag{2}$$

where 0_2 and I_2 are the 2-by-2 null and identity matrices, respectively, $\Lambda \in \mathbb{R}^{2\times 2}$ quantifies the mean-reversion effect, while Ω represents the process noise. If Λ has positive and distinct eigenvalues, Λ can be written as $\Lambda = \bar{G}\Gamma\bar{G}^{-1}$, where $\Gamma = \text{diag}(\gamma)$. The target state evolution is given by the first moment of the solution of (1), which takes the form

$$\begin{aligned} x_k = {} & G\tilde{\Phi}(t_k - t_{k-1}, \gamma)G^{-1}x(t_{k-1}) \\ & + G\tilde{\Psi}(t_k - t_{k-1}, \gamma)\bar{G}^{-1}u + w_k, \end{aligned} \tag{3}$$

where $G = I_2 \otimes \bar{G}$, with $\otimes$ denoting the Kronecker product. The full expressions of $\tilde{\Phi}(t, \gamma)$ and $\tilde{\Psi}(t, \gamma)$ can be found in [10], [11].

Clearly, (3) is suitable to represent a non-maneuvering object, whose long-run mean velocity does not change over time. However, the model can be easily extended to model linear piecewise trajectories (such as a ship that navigates by following a sequence of way-points) where each segment is characterized by a given configuration of kinematic parameters that are piecewise-constant functions of time.

2.2. Nominal/anomalous object motion

Based on dynamic model (3) described by a mean-reverting OU stochastic process, the anomaly can be represented by an object that deviates from a preset route by changing its nominal mean velocity. Moreover, using a piecewise OU model, the specific mode of motion $\nu_k = i$ of the object within a given time interval $[t_{k-1}, t_k]$ can be specified by the other two parameters of the process: the reversion rate Λ_k^i and the noise covariance Ω_k^i characterizing mode i.

Under such assumptions, the target under nominal or anomalous behavior can be modeled by

$$x_{k+1} = \begin{cases} \Phi_k\, x_k + \Psi_k\, d_k + w_{k+1}, & \text{under } H_0 \\ \Phi_k\, x_k + \Psi_k\, d_k + \bar{\Psi}_k\, u_k + w_{k+1}, & \text{under } H_1 \end{cases} \tag{4}$$

where: k is the time index; $x_k \in \mathbb{R}^m$ is the object kinematic state; $d_k \in \mathbb{R}^q$ is a known input characterizing a target under hypothesis H_0, i.e., that follows its nominal trajectory (d_k is the object's nominal long-run mean velocity in the time interval $[t_{k-1}, t_k]$); $u_k \in \mathbb{R}^q$ is an unknown input affecting the object dynamics in the time window $[t_{k-1}, t_k]$ when the object behavior is under hypothesis H_1, i.e., when the target is deviating from its nominal path; $\nu_k \in \mathcal{M} = \{1, 2, ..., \mu\}$ is the mode in operation at time k; $\delta_k = t_k - t_{k-1}$; $\Phi_k \triangleq \Phi(\nu_k, \delta_k)$ is a known mode-matched state transition matrix describing the state evolution of the object under a specific mode ν_k; $\Psi_k \triangleq \Psi(\nu_k, \delta_k)$ and $\bar{\Psi}_k \triangleq \Psi(\nu_k, \delta_k)$ are two mode-matched input matrices; w_k is a random process disturbance with probability density function (PDF) $p_w(\nu_k, \cdot)$. Note that the unknown mean velocity term appearing in (4) under H_1 is treated, differently from the deterministic parameter u in (1), as a stochastic process $\{u_k\}$, independent of x_0, $\{w_{k+1}\}$ and $\{v_k\}$. For the observation model, we consider the *measurement set* $\mathcal{Z}_k = \mathcal{C}_k \cup \mathcal{Y}_k$, where $\mathcal{C}_k$ is the finite set of spurious observations, while $\mathcal{Y}_k$ is the set of target-originated measurements with y_k received with probability $p_d \in (0, 1]$, and given by

$$y_k = C_k x_k + v_k \tag{5}$$

where C_k is a known measurement function and v_k is a random measurement noise.

3. JOINT ANOMALY DETECTION AND TRACKING

In this section, we review the notion of *Hybrid Bernoulli Random Set (HBRS)*, that is a special type of RFS introduced by [14–18] in the context of Bayesian state estimation with switching unknown inputs affecting both the dynamic and the measurement model. The resulting hybrid Bernoulli state-space model allows us to frame and solve the problem of joint anomaly detection and tracking in a random set-based Bayesian framework. Following this approach, it is possible to obtain an exact recursion in terms of integral equations that generalize the Bayes and Chapman-Kolmogorov equations used for the solution of joint input-and-state estimation [21], [22] (i.e., when the unknown input is not switching), and standard Bernoulli filtering [23–25] (i.e., for a system without unknown inputs).

3.1. Hybrid Bernoulli state-space model

The anomalous behavior of the object under tracking can be represented by means of a switching unknown input, i.e. an *unknown velocity* Bernoulli RFS $\mathcal{U}_k \in \mathcal{B}(\mathbb{U})$, where $\mathcal{B}(\mathbb{U}) = \emptyset \cup \mathcal{S}(\mathbb{U})$ is a set of all finite subsets of the velocity space $\mathbb{U} \subseteq \mathbb{R}^q$, while $\mathcal{S}$ denotes the set of all singletons $\{u\}$ such that $u \in \mathbb{U}$. Thus, we can model the unknown velocity input $\mathcal{U}_k$, at time k, as a finite set that can take on either the empty set, when the object dynamics is under H_0 at time k, or a singleton $\{u_k\}$ otherwise, i.e.

$$\mathcal{U}_k = \begin{cases} \emptyset, & \text{under } H_0 \\ \{u_k\}, & \text{under } H_1. \end{cases} \tag{6}$$

If we further denote the Euclidean space for the object state vector with $\mathbb{X} \subseteq \mathbb{R}^m$, then it is possible to define a new state variable $(\mathcal{U}, x)$ on the hybrid space $\mathcal{B}(\mathbb{U}) \times \mathbb{X}$. This special random set, referred to as *hybrid Bernoulli random set* in [14–18], combines the Bernoulli RFS $\mathcal{U}$, as well as the random state vector x. An HBRS on $\mathcal{B}(\mathbb{U}) \times \mathbb{X}$ is fully specified by (r, f^0, f^1): with probability $1 - r$, the object is under nominal behavior ($\mathcal{U} = \emptyset$) with kinematic state distributed according to a PDF $f^0(x)$ defined on $\mathbb{X}$; with probability r, $\mathcal{U}$ is a singleton and the joint input-state variable is distributed according to a joint PDF $f^1(u, x)$ defined on $\mathbb{U} \times \mathbb{X}$. The HBRS $(\mathcal{U}, x)$ can be corrected and predicted in a recursive fashion so as to form the *hybrid Bernoulli filter*.

3.2. The hybrid Bernoulli filter

Consider the system (4)-(5) with no direct feedthrough of the unknown velocity input u_k to the output y_k. In this case, $\mathcal{U}_k$ must be estimated with one step delay, since $\mathcal{Z}_{k+1}$ is the first measurement set containing information on $\mathcal{U}_k$. Hence, the HBF will sequentially update the hybrid Bernoulli density $f(\mathcal{U}_{k-1}, x_k|\mathcal{Z}^k)$ of the unknown velocity set $\mathcal{U}_{k-1}$ and state x_k conditioned on all the information available up to time k. Let $f(\mathcal{U}_{k-1}, x_k|\mathcal{Z}^{k-1})$ be the prior density at time k. Then, given the measurement set $\mathcal{Z}_k$, the posterior density at time k can be obtained by means of the Bayes rule

$$f(\mathcal{U}_{k-1}, x_k|\mathcal{Z}^k) = \frac{\eta(\mathcal{Z}_k|x_k)\, f(\mathcal{U}_{k-1}, x_k|\mathcal{Z}^{k-1})}{\iint \eta(\mathcal{Z}_k|x_k)\, f(\mathcal{U}_{k-1}, x_k|\mathcal{Z}^{k-1})\, \delta\mathcal{U}_{k-1} dx_k}. \tag{7}$$

The prior density at time $k+1$ can be computed via the Chapman-Kolmogorov equation

$$f(\mathcal{U}_k, x_{k+1}|\mathcal{Z}^k) = \iint \phi(\mathcal{U}_k, x_{k+1}|\mathcal{U}_{k-1}, x_k)\, f(\mathcal{U}_{k-1}, x_k|\mathcal{Z}^k)\, \delta\mathcal{U}_{k-1} dx_k \tag{8}$$

where the set integral over the hybrid state space is defined as

$$\int f(\mathcal{U}, x)\delta\mathcal{U}\, dx = \int f(\emptyset, x)\, dx + \iint f(\{u\}, x)\, du\, dx. \tag{9}$$

Exact prediction and correction equations of the hybrid Bernoulli filter with no direct feedthrough can be obtained by substituting the transition density $\phi(\mathcal{U}_k, x_{k+1}|\mathcal{U}_{k-1}, x_k)$ and the likelihood function $\eta(\mathcal{Z}_k|x_k)$ into (7)-(8) (see [9], [15]). An analytical solution of the hybrid Bernoulli filtering problem in the case of direct feedthrough of the unknown input into the output can be found in [14], [15].

4. ADAPTIVE ANOMALY DETECTION AND TRACKING WITH UNKNOWN PARAMETERS

In order to handle switching parameters that can change the dynamic model in effect for the object's motion, single-model approaches like the one proposed in [9] need to be made to accommodate switched stochastic systems. To this end, the idea is to rely on the multiple-model (MM) approach [26]. The true mode of the object is, thus, supposed to switch according to a homogeneous Markov chain with known transition probabilities $\pi_{ji} = \text{prob}\,(\nu_k = i \,|\, \nu_{k-1} = j)$, $i, j \in \mathcal{M}$, $\sum_{i=1}^{\mu} \pi_{ji} = 1$. The goal becomes to solve the problem of anomaly detection and simultaneous state and mode estimation. This amounts to jointly estimating, at each time k, the velocity Bernoulli RFS $\mathcal{U}_k$, the state x_k, and the mode ν_k representing specific OU parameters, given the set of measurements $\mathcal{Z}^k \triangleq \cup_{i=1}^k \mathcal{Z}_i$ up to time k.

4.1. Multiple-model adaptive hybrid Bernoulli filter

For state estimation of jump Markov systems modeled by (4), the HBRS defined in Section 3.1 needs to be augmented so as to include the hidden mode of motion ν in the new state variable $(\mathcal{U}, x, \nu)$. This is referred to as *Multiple Model Hybrid Bernoulli Random Set* (MM-HBRS), and takes values in $\mathcal{B}(\mathbb{U}) \times \mathbb{X} \times \mathcal{M}$. An MM-HBRS is fully specified by the probability r of $\mathcal{U}$ being a singleton, the mode-conditioned PDF $f^0(x, \nu)$, and the mode-conditioned joint PDF $f^1(u, x, \nu)$, i.e. by the density

$$f(\mathcal{U}, x, \nu) = \begin{cases} (1-r)\, f^0(x, \nu), & \text{if } \mathcal{U} = \emptyset \\ r \cdot f^1(u, x, \nu), & \text{if } \mathcal{U} = \{u\} \end{cases} \tag{10}$$

with the following integration over the new state space:

$$\sum_{i=1}^{\mu} \rho^i \int_{\mathcal{B}(\mathbb{U})\times\mathbb{X}} f(\mathcal{U}, x|\nu = i)\, \delta\mathcal{U}\, dx \tag{11}$$

where $\int f(\mathcal{U}, x|\nu = i)\, \delta\mathcal{U}\, dx$ is given by (9), and $\rho^i \triangleq \text{prob}(\nu = i|\mathcal{Z})$ is the mode probability of mode i, given the measurement set $\mathcal{Z}$. Notice that in (11) $f(\mathcal{U}, x, \nu)$, which will be referred to as *multiple model hybrid Bernoulli density*, integrates to one given that i) integration with respect to $\mathcal{U}$ and x equals 1, $f^0(x)$ and $f^1(u, x)$ being conventional PDFs, and ii) $\sum_{i=1}^{\mu} \rho^i = 1$. Similar to the single-model HBF described in Section 3.2, a MM-HBRS can be corrected and predicted in a recursive fashion so as to form a novel *Multiple-Model Adaptive Hybrid Bernoulli Filter* (MMA-HBF).

For the dynamic model, let us assume that i) a target under nominal behavior at time k will start deviating during the sampling interval δ_{k+1} with probability p_b; ii) if the target is already deviating at time k, the anomalous deviation will carry on from k to $k+1$ with probability p_s. It is further assumed that $(\mathcal{U}, x, \nu)$ is a Markov process with joint transitional density

$$\begin{aligned}\phi(\mathcal{U}_k, x_{k+1}, \nu_{k+1}|\mathcal{U}_{k-1}, x_k, \nu_k) \\ = \phi(x_{k+1}, \nu_{k+1}|\mathcal{U}_k, x_k, \nu_k)\, \phi(\mathcal{U}_k|\mathcal{U}_{k-1}).\end{aligned} \tag{12}$$

In addition, note that

$$\begin{aligned}&\phi(x_{k+1}, \nu_{k+1}|\mathcal{U}_k, x_k, \nu_k) \\ &= \begin{cases} \phi(\nu_{k+1}|\nu_k)\, \phi(x_{k+1}|x_k, \nu_k), & \text{if } \mathcal{U}_k = \emptyset \\ \phi(\nu_{k+1}|\nu_k)\, \phi(x_{k+1}|u_k, x_k, \nu_k), & \text{if } \mathcal{U}_k = \{u_k\} \end{cases}\end{aligned} \tag{13}$$

and

$$\phi(\mathcal{U}_k|\emptyset) = \begin{cases} 1 - p_b, & \text{if } \mathcal{U}_k = \emptyset \\ p_b\, g(u_k), & \text{if } \mathcal{U}_k = \{u_k\} \end{cases} \tag{14}$$

$$\phi(\mathcal{U}_k|\{u_{k-1}\}) = \begin{cases} 1 - p_s, & \text{if } \mathcal{U}_k = \emptyset \\ p_s\, \phi(u_k|u_{k-1}), & \text{if } \mathcal{U}_k = \{u_k\} \end{cases} \tag{15}$$

where $g(u_k)$ is the initial guess PDF of the unknown velocity input. The measurement model follows a single-target in clutter model [9] with likelihood

$$\eta(\mathcal{Z}_k|x_k) = \psi(\mathcal{Z}_k) \left[1 - p_d + p_d \sum_{y_k \in \mathcal{Z}_k} \frac{\ell(y_k|x_k)}{\lambda_k\, \kappa(y_k)}\right] \tag{16}$$

where $\ell(y_k|x_k)$ is the likelihood of the object-generated measurement, $\kappa(\cdot)$ describes the distribution of clutter, λ_k is the number of received observations at time k, and $\psi(\cdot)$ is the FISST probability density of spurious-only measurements. Using the above dynamical and measurement models, exact prediction and, respectively, correction equations for the proposed MMA-HBF (here omitted due to lack of space) can be easily obtained.

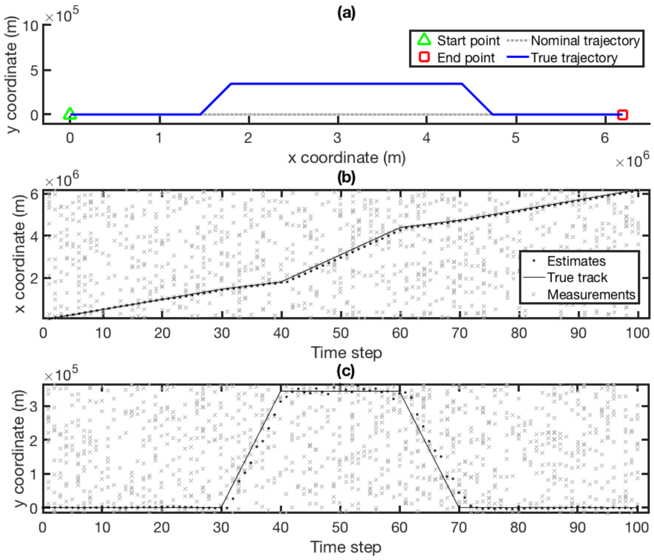

Fig. 1: (a) Nominal and true object trajectories. (b)-(c) Measurement data and true/estimated object positions.

5. SIMULATION RESULTS

The performance of the proposed MMA-HBF, implemented via Gaussian mixtures, has been tested on a simulated scenario concerning maritime traffic monitoring. The objective is to detect and track route deviations of a target navigating along a nominal path with cruise velocity $d = [3, 0]^T m/s$, while estimating the mode of motion at each time instant. The evolution of the object state over time is assumed to follow the piecewise OU model (4), where each mode of motion corresponds to one set of OU parameters $\Theta_j = \{\gamma_j, \sigma_j\}$, $j = 1, \ldots, \mu$ such that $\Gamma_j = \text{diag}(\gamma_j)$ and $\Omega_j \Omega_j^T = \sigma_j I$. In particular, we considered $\Theta_1 = \{0.1, 0.01\}$, $\Theta_2 = \{0.001, 0.2\}$, and $\Theta_3 = \{0.01, 3\}$, where Θ_1 is the true mode. Moreover, we used $\bar{\Psi}_k = \Psi_k$, $p_d = 0.98$, and $\delta = 5$ h. For the filter, we set $p_b = 0.001$, $p_s = 0.8$, $r_{1|0} = 0.5$, $\hat{x}_{1|0} = [10, 10, 0, 3],^T$ $P_{1|0} = \text{diag}(10^3, 10^3, 10^2, 10^2)$, and $\tilde{u}_{1|0} = [2, 2]^T$, $\tilde{P}^u_{1|0} = 10^3\, I_2$ as mean and covariance of the a priori PDF $g(u_k)$. The object-generated measurements follow model (5) where $v_k \sim \mathcal{N}(\cdot; 0, R)$, with $R = \text{diag}(50^2, 50^2, 10, 10)$, while clutter is modeled as a Poisson RFS with an average number of 10 returns per scan and a uniform spatial distribution over the observation region. The simulation is run over 100 time steps, where the object deviation from the nominal trajectory is initiated at time step $k = 31$ and terminated at time step $k = 70$. Fig. 1 (a) shows the nominal and the true trajectory, while Figs. 1 (b)-(c) show the state estimation performance of the filter for a single realization. It can be seen from Fig. 2 (a) that the MMA-HBF promptly detects the start and the end of the anomaly as soon as the target starts deviating and, respectively, once it moves back to its nominal trajectory. Fig. 2 (b) shows the mode probabilities computed at each time step. The filter detects the true object's mode of motion (mode 1), i.e. it can estimate the unknown mean-reverting rate and noise level parameters of the OU dynamic model. The estimated long-run mean velocity is illustrated in Fig. 3. The proposed filter provides accurate velocity estimates under both nominal and anomalous behavior, albeit slightly degraded in the latter.

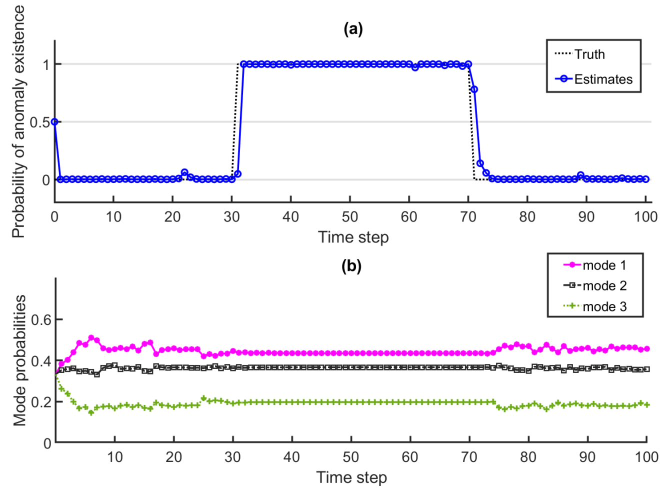

Fig. 2: (a) Anomaly detection performance and (b) mode probabilities vs. time.

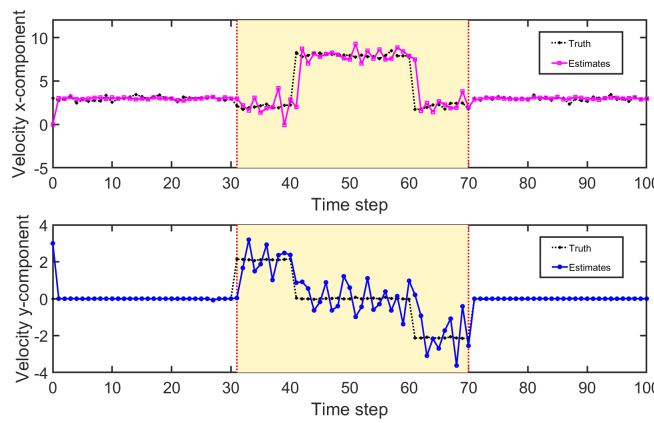

Fig. 3: Performance in terms of long-run mean velocity reconstruction under nominal and anomalous object behavior. The anomalous activity takes place between time step 31 and 70 (this time window is highlighted by a yellow band).

6. CONCLUSIONS

This work considered the use of a multiple-model adaptive hybrid Bernoulli filter (MMA-HBF) for joint anomaly detection and trajectory estimation of a single target whose motion is described by mean-reverting OU stochastic processes with unknown parameters. The proposed adaptive filter was derived using finite set statistics and the multiple-model approach. Simulation results show that the MMA-HBF is a promising candidate for anomaly detection and target tracking with unknown parameters of the underlying dynamic model. Future research will focus on validating the MMA-HBF in real-world examples of maritime surveillance.

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